Spillover Alleviation for Nonlinear Active Control of Vibration

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The paper discusses the application of nonlinear saturation controls based on the second method of Lyapunov to the active control of vibration of linear structures. The spillover instability associated with the neglected dynamics is demonstrated. A spillover alleviation procedure is presented and demonstrated. It consists of a local modification of the control in the vicinity of the equilibrium, which makes the composite system (controlled modes-observer-neglected dynamics) benefit from the inherent stability properties of the open loop system.

I. Introduction

IN the design of control systems for reducing structural vibrations, performance requirements often lead to high bandwidth control systems. For various practical reasons, the control system must be designed with a reduced order model of the plant (usually including only a few low frequency structural modes). The interaction between the control system and the modes that have not been taken into account in its design (the residual modes) is known as the spillover effect. The residual modes usually have a small stability margin provided by structural damping; they can be made unstable by spillover. This has been pointed out as one of the major issues in the active control of large space structures.[1]

The spillover mechanism is illustrated in Fig. 1, where the flexible system reacts through the controlled and residual modes. The residual modes are excited by the control (control spillover) and contribute to the output signal fed into the observer (observation spillover). When both control and observation spillover are present, they may lead to instability in the residual modes.[2,3]

Various ways of alleviating spillover have been proposed: 1) increase passive damping,[4] 2) supplement the modal control by direct velocity feedback with colocated sensors and actuators,[5] 3) include in the control system performance index a penalty for the control spillover, thus forcing the control to be "orthogonal" to the residual modes[6,7] (the idea applies to observation spillover as well, by duality), and 4) use frequency dependent weighting matrices to increase the penalty for the high frequency components of the control.[8] (Most of the literature applies to linear control.)

When the control force is subject to amplitude constraints, nonlinear controls minimizing a quadratic performance index can be constructed using the second method of Lyapunov.[9,10] The robustness of the resulting controls vis a vis regular perturbations has been exploited to synthesize controls for uncertain systems.[11-14]

The present study was initiated to investigate how saturation controls based on the second method of Lyapunov would behave when applied to the control of vibration of large space structures, where a critical part of the system uncertainty consists of singular perturbations (controller designed with a reduced order model of the plant) and the neglected dynamics has a small stability margin provided by the structural damping. It was found that when the controlled states are near equilibrium the sensor output is dominated by the residual modes and the closed loop system experiences a strong spillover instability. This instability can be alleviated by a local modification of the control in the vicinity of the equilibrium, which reinstates the asymptotic stability properties of the open loop system. The distance from the equilibrium is measured by a positive definite function $V(x_c)$ of the controlled states (e.g., the total energy in the controlled modes). When the controlled states enter a neighborhood δ of the equilibrium, the magnitude of the control is modified according to the ratio $V(x_c)/\delta$. Local stability results can be established, with or without regular perturbations. The compromise between performance and stability domain depends on δ and the quality of the estimate of $V(x_c)$ that can be obtained.

Saturation Control

Consider the linear time invariant asymptotically stable system

$$\dot{x} = Ax + Bu \tag{1}$$

and the quadratic performance index

$$J = \int_0^\infty x' Qx \mathrm{d}t \tag{2}$$

If the magnitude of the control components are subject to the constraint

$$|u_i| \leq a_i \tag{3}$$

The components of the optimal control are given by[9,10]

$$u_i = -a_i \operatorname{sign}(B'Px)_i \tag{4}$$

where P is solution of the Lyapunov equation

$$PA + A'P + Q = 0 \tag{5}$$

Control (4) is the one leading to the most negative value of the derivative of the Lyapunov function $V(x) = x'Px$.

Now, consider the regularly perturbed system

$$\dot{x} = (A + \Delta A)x + (B + \Delta B)u + Hv(t) \tag{6}$$

where the system matrix uncertainty ΔA, input matrix uncertainty ΔB, and unceraint in the input Hv are assumed to

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satisfy the matching condition that can be written as

$$\Delta A = BD, \quad \Delta B = BE, \quad H = BF \tag{7}$$

(which physically means that the uncertainty is within the range of the input). It follows from Eq. (7) that Eq. (6) can be transformed into

$$\dot{x} = Ax + Bu + Be(x,t) \tag{8}$$

where

$$e(x,t) \triangleq Dx + Eu + Fv$$

is the lumped uncertainty. If a norm $\rho(x)$ can be found for $e(x,t)$ [vector norms are Euclidean, and the corresponding matrix norms are $\|A\|^2 = \lambda_{\max}(A'A)$]:

$$\|e(x,t)\| \leq \rho(x) \tag{9}$$

the following control[11-14] guarantees stability and uniform ultimate boundedness within a neighborhood of the equilibrium of the system:

$$u = \begin{cases} -\dfrac{B'Px}{\|B'Px\|}\rho(x) & \text{if } \|B'Px\| > \epsilon \quad (10a) \\ -\dfrac{B'Px}{\epsilon}\rho(x) & \text{if } \|B'Px\| \leq \epsilon \quad (10b) \end{cases}$$

where ϵ is a prescribed positive constant and P the solution of Eq. (5). The reason for using $\epsilon > 0$ is to avoid the mathematical complications associated with discontinuous differential equations. The same modification can be applied to Eq. (4).

If upper bounds can be found on $\|D\|$, $\|E\|$, and $\|Fv\|$, and if max $\|E\| < 1$ (which, physically, means that the uncertainty in the input matrix is not too large, so that the control acts in the desired direction), it can be shown[14] that an upper bound for $\rho(x)$ is given by

$$\rho(x) \leq \bar{\rho}(x) \triangleq \lambda_1 \|x\| + \lambda_2$$

where

$$\lambda_1 = \frac{\max\|D\|}{1 - \max\|E\|} \quad \text{and} \quad \lambda_2 = \frac{\max\|Fv\|}{1 - \max\|E\|}$$

Controls (4) and (10) are essentially similar. In the single input case as in the example to be considered later, Eqs. (4) and (10a) are identical, except that the magnitude of control (10) depends on the state if there are system uncertainties.

Application to Vibration Control

For a linear system, the fact that P is a solution of the Lyapunov equation (5) indicates that $x'Px$ is a Lyapunov function for the system under consideration. For a damped vibrating structure without rigid body modes, the total energy in the system (strain + kinetic) is a Lyapunov function, and Eq. (5) need not be solved to construct the control (10). (Although the corresponding Q matrix is not positive definite, the error criterion does not vanish identically along any trajectory because of the structural damping.) After a few algebraic manipulations it is readily obtained that, with that particular choice of a Lyapunov function, control (10) can be written as

$$u = \begin{cases} -\dfrac{v_a}{\|v_a\|}\rho(x) & \text{if } \|v_a\| > \epsilon \quad (11a) \\ -\dfrac{v_a}{\epsilon}\rho(x) & \text{if } \|v_a\| \leq \epsilon \quad (11b) \end{cases}$$

where v_a is the vector containing the velocities at the actuators location, for the nominal system. Note that if v_a is available, control (11) cannot feed energy into the system, for $v_a'u \leq 0$. This would be the case if colocated actuators and sensors were used. In general, the control law will be implemented with the velocity v_a calculated from the reconstructed state obtained from an observer that is affected by the neglected dynamics.

Of course, any other P solution of Eq. (5) for any particular choice of a positive definite matrix Q (e.g., $Q = I$) is suitable in implementing control (10).

Local Stability Conditions, Spillover Alleviation

Using the subscripts c and r to refer to the controlled and residual modes, respectively, the (nominal) plant is governed by

$$\dot{x}_c = A_c x_c + B_c u \tag{12}$$

$$\dot{x}_r = A_r x_r + B_r u \tag{13}$$

The sensor output is

$$y = C_c x_c + C_r x_r \tag{14}$$

The observer state $\hat{x}_c$ is governed by

$$\dot{\hat{x}}_c = A_c \hat{x}_c + B_c u + K_c(y - C_c \hat{x}_c) \quad \hat{x}_c(0) = 0 \tag{15}$$

The controller operates on the reconstructed state $u = u(\hat{x}_c)$. Introducing the estimator error $e_c = \hat{x}_c - x_c$, it is readily obtained, using Lyapunov's first method,[15] that the composite closed loop system (controlled modes/observer/residual modes) is stable in the vicinity of the origin if the following linearized system is stable

$$\begin{bmatrix} \dot{x}_c \\ \dot{e}_c \\ \dot{x}_r \end{bmatrix} = \begin{bmatrix} A_c + B_c\left(\dfrac{\partial u}{\partial x_c}\right)_0 & B_c\left(\dfrac{\partial u}{\partial e_c}\right)_0 & 0 \\ 0 & A_c - K_c C_c & K_c C_r \\ B_r\left(\dfrac{\partial u}{\partial x_c}\right)_0 & B_r\left(\dfrac{\partial u}{\partial e_c}\right)_0 & A_r \end{bmatrix} \begin{bmatrix} x_c \\ e_c \\ x_r \end{bmatrix} \tag{16}$$

If a linear state feedback $u = G_c \hat{x}_c$ is used, $(\partial u/\partial e_c)_0 = (\partial u/\partial x_c)_0 = G_c$ and Eq. (16) is reduced to the classical result.[2]

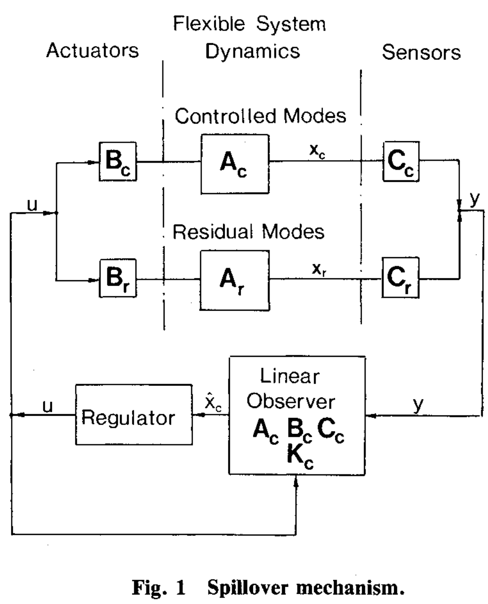

Fig. 1 Spillover mechanism.

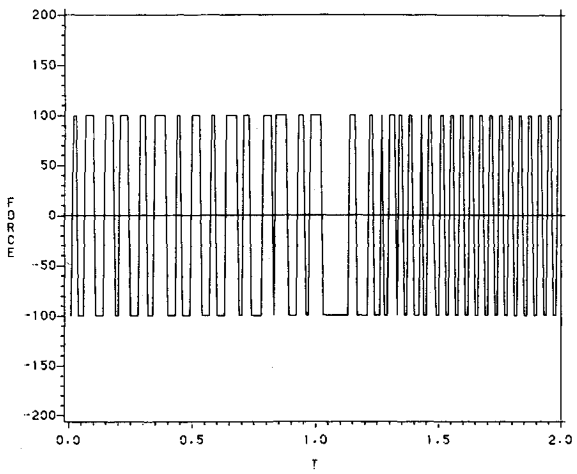

Fig. 2 Saturation control, $\rho=100$; $\epsilon=0.5$. control force.

If a control can be constructed which satisfies

$$\left(\frac{\partial u}{\partial x_c}\right)_0 = \left(\frac{\partial u}{\partial e_c}\right)_0 = 0 \tag{17}$$

the eigenvalues of Eq. (16) become uncoupled and are those of A_c, $A_c - K_c C_c$, and A_r, respectively. Consequently, the closed loop system has, in the vicinity of the origin, the same stability properties as the open loop system. This is the basis of the spillover alleviation procedure proposed in this study: The control is modified in the vicinity of the equilibrium state $x_c = 0$ according to

$$\begin{aligned} u^*(x_c) &= u(x_c) & V(x_c) \geq \delta \\ &= u(x_c) \cdot \frac{V(x_c)}{\delta} & V(x_c) < \delta \end{aligned} \tag{18}$$

where $V(x_c)$ is a continuously differentiable locally positive definite function of the controlled states, and δ is a small positive constant defining the vicinity of the equilibrium state $x_c = 0$. [$V(x)$ satisfies $V(0) = 0$ and $V(x) > 0$ for all $x \neq 0$ belonging to some ball $B_r = \{x: \|x\| \leq r, r > 0\}$ (see Ref. 15, p. 141). This is sufficient if $u(0) = 0$. If not, $V(x)$ will be required to satisfy the additional condition $(\partial V(x)/\partial x)_0 = 0$.] Of course, since the actual state x_c is not known, the modified control u^* will be implemented with the reconstructed state, $u^* V(\hat{x}_c)$.

Example

We consider the simply supported beam considered in Refs. 2 and 3 with an actuator located at 1/6 of the length and a displacement transducer located at 5/6 of the length. The first three modes are controlled and the spillover due to the fourth mode is investigated. Control (11) is implemented on the reconstructed state obtained by the same full state Luenberger observer, as in Refs. 2 and 3). The following values have been adopted for the parameters: $\rho = 100$ and $\epsilon = 0.5$ (this value of ρ makes the decay rate comparable to that of the linear example of Refs. 2 and 3). Figures 2 and 3 give the time histories of the control force and the sensor displacement when the beam is released from rest with a unit initial amplitude in the first three modes. No structural damping is assumed. Figure 4 gives the time history of the total energy in the controlled modes and in the residual mode.

The three figures indicate a spillover instability. After a while, the vibration of the beam is dominated by the fourth mode, which is much more unstable than in the linear example considered in Refs. 2 and 3. If some structural damping is added, the response is unstable (in the sense of Lyapunov) but bounded. The system goes into a limit cycle in the residual mode. For this particular example, it is only when the damping ratio of the fourth mode becomes larger than $\xi = 0.06$ that the composite system is asymptotically stable.

Examining Figs. 2–4, we observe that when the total energy (kinetic + potential) in the controlled modes has dropped, the control continues feeding energy in the residual mode, thus leading to instability. This suggests using the total energy in the controlled modes (or any positive definite function of the controlled states) to taper off the control when the controlled modes have been brought back to the appropriate vicinity of the equilibrium state $x_c = 0$. This observation was the starting point for constructing the new control (18).

For the previous example, taking the total energy as $V(x)$ and $\delta = 100$, the new control leads to the results of Figs. 5–7 ($\xi = 0$) which are directly comparable with Figs. 2–4. The closed loop system can be stabilized with a minute amount of damping in the residual mode. The improvement is achieved without affecting the overall performance of the control $u(x_c)$ as can be seen by comparing the full line curves in Figs. 8a–8d.

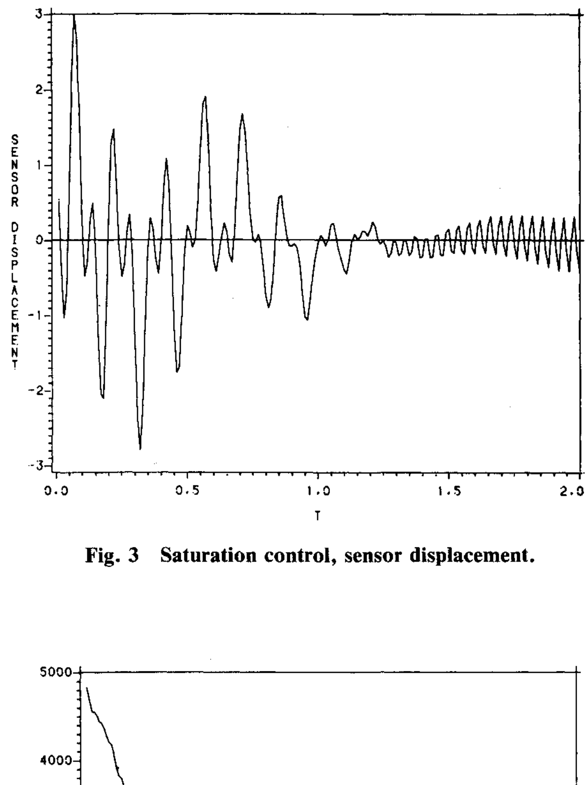

Fig. 3 Saturation control, sensor displacement.

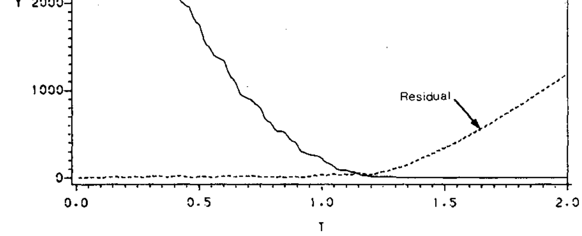

Fig. 4 Saturation control, energy in the controlled modes and in the residual mode.

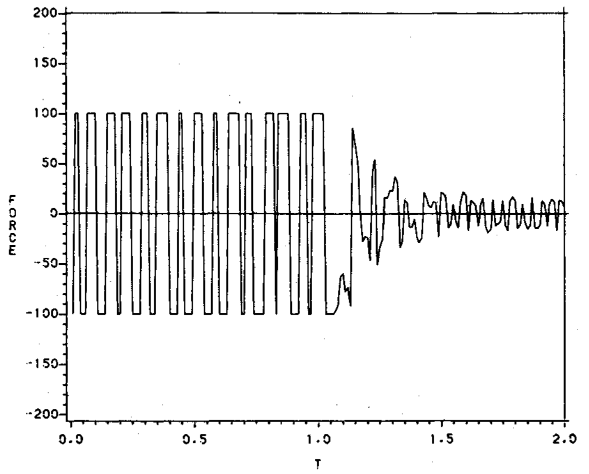

Fig. 5 Modified control, control force.

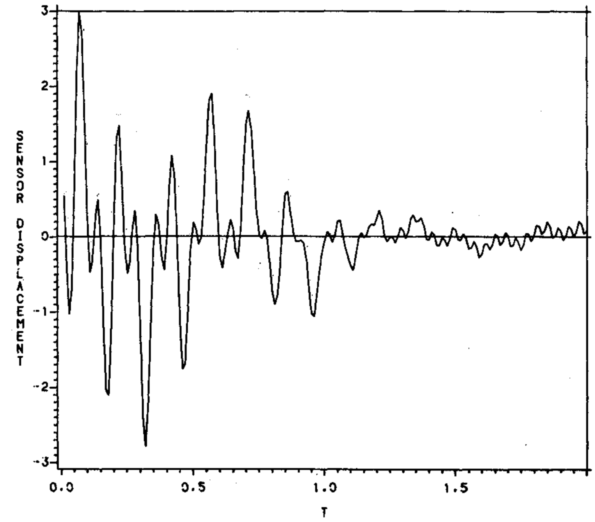

Fig. 6 Modified control, sensor displacement.

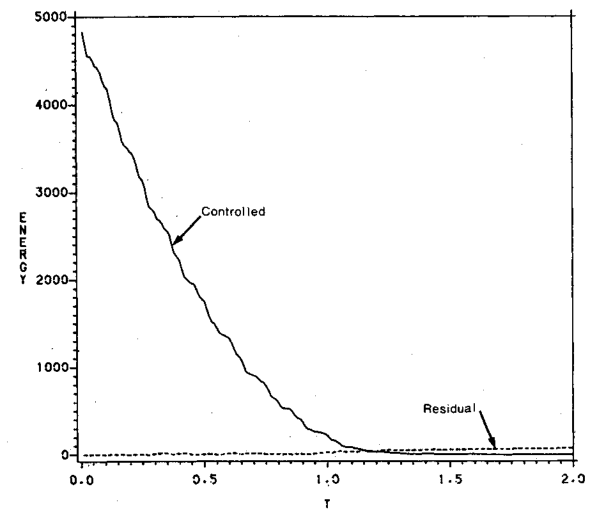

Fig. 7. Modified control, energy in the controlled modes and in the residual mode.

Effect of Modeling Error

If there are uncertainties in the system parameters, the following equations must replace Eqs. (12–14) for the description of the actual plant:

$$\dot{x}_c = (A_c + \Delta A)x_c + (B_c + \Delta B)u \tag{19}$$

$$\dot{x}_r = A_r x_r + B_r u \tag{20}$$

$$y = (C_c + \Delta C)x_c + C_r x_r \tag{21}$$

where ΔA, ΔB, and ΔC represent the uncertainty in the modeling of the controlled modes. (No uncertainty needs to be included for the residual modes that are not part of the modeling.) u is the only nonlinear term in the equation of the composite system. Again, if u is continuously differentiable in the vicinity of the origin, using Lyapunov's first method one finds that the closed loop system is asymptotically stable if the eigenvalues of

$$\begin{bmatrix} A_c + \Delta A + (B_c + \Delta B)\left(\dfrac{\partial u}{\partial x_c}\right)_0 & (B_c + \Delta B)\left(\dfrac{\partial u}{\partial e_c}\right)_0 & 0 \\ -\Delta A + K_c \Delta C - \Delta B\left(\dfrac{\partial u}{\partial x_c}\right)_0 & A_c - K_c C_c - \Delta B\left(\dfrac{\partial u}{\partial e_c}\right)_0 & K_c C_r \\ B_r\left(\dfrac{\partial u}{\partial x_c}\right)_0 & B_r\left(\dfrac{\partial u}{\partial e_c}\right)_0 & A_r \end{bmatrix} \tag{22}$$

have negative real parts. Because of Eq. (17), this condition is satisfied providing the open loop system is asymptotically stable. Thus, in spite of the uncertainty in the modeling of the controlled modes and the presence of residual modes, the closed loop system will inherit the stability properties of the open loop system.

In active control of vibration, the uncertainties ΔA, ΔB, ΔC include errors in the mode shapes, natural frequencies, and modal damping. However, since the open loop system is guaranteed to be stable (with and without uncertainty), the closed loop system is also guaranteed to be stable, locally. Note that in this section, neither matching condition nor any bound on the magnitude of the uncertainty were assumed.

Effect of Observer Errors

The stability results established in the previous sections are local. One may consequently wonder about the behavior of the control when there is a significant discrepancy between the reconstructed state $\hat{x}_c$ and the actual state x_c. In this situation $V(\hat{x}_c)$ can be considerably different from $V(x_c)$. This is illustrated in Fig. 9 which shows for $\delta = 100$ and $\xi = 0.005$ (same as Fig. 8d) the ratios $V(x_r)/V(x_c)$ and $V(\hat{x}_c)/V(x_c)$ where

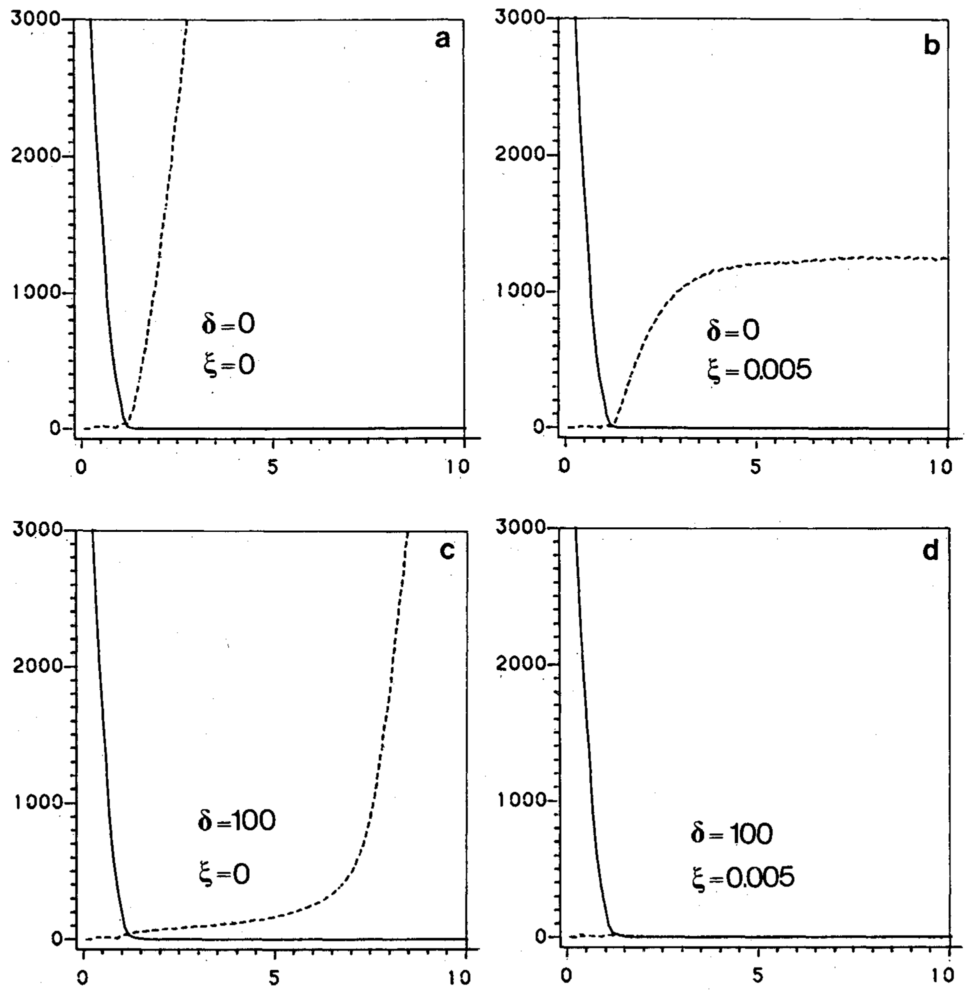

Fig. 8 Total energy in the controlled and residual modes as functions of time, for various values of δ and ξ: —controlled modes, ---residual modes.

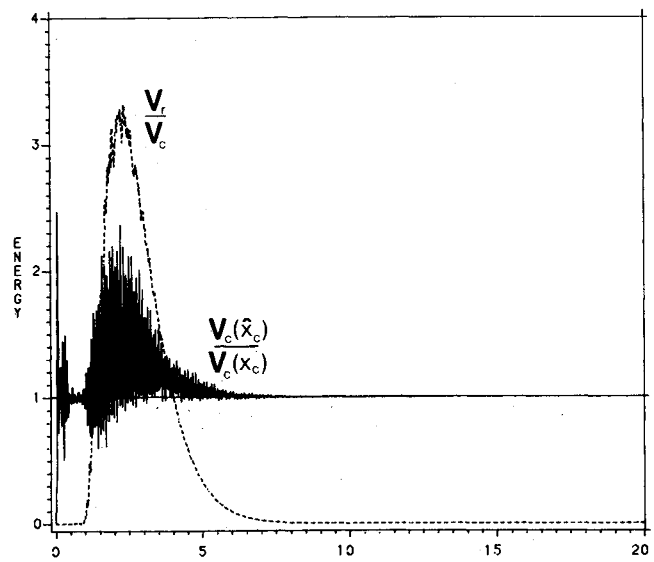

Fig. 9 $V(x_r)/V(x_c)$ and $V(\hat{x}_c)/V(x_c)$ as functions of time, free response for $\delta = 100$ and $\xi = 0.005$.

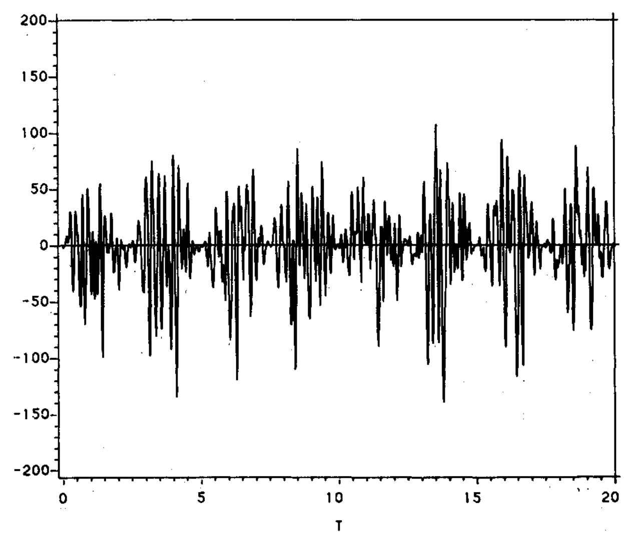

Fig. 10 Broadband modulated noise excitation.

$V(x_c)$, $V(x_r)$, and $V(\hat{x}_c)$ are, respectively, the actual energy in the controlled modes, the energy in the residual mode, and the estimated energy in the controlled modes based on the reconstructed state. It can be seen that, except for the transient due to the zero initial conditions of the observer, $V(\hat{x}_c)$ is a good estimate of $V(x_c)$ as long as $V(x_r)$ is small as compared to $V(x_c)$.

When $V(\hat{x}_c)$ can depart considerably from $V(x_c)$, one faces the usual compromise between performance and stability: the larger δ (defining the vicinity of the equilibrium where the control is altered), the larger the stability domain but the lesser the performance. It is likely that an improved estimate of $V(x_c)$ can be devised to take advantage of the slow variation of the total energy. This is the topic of further investigations.

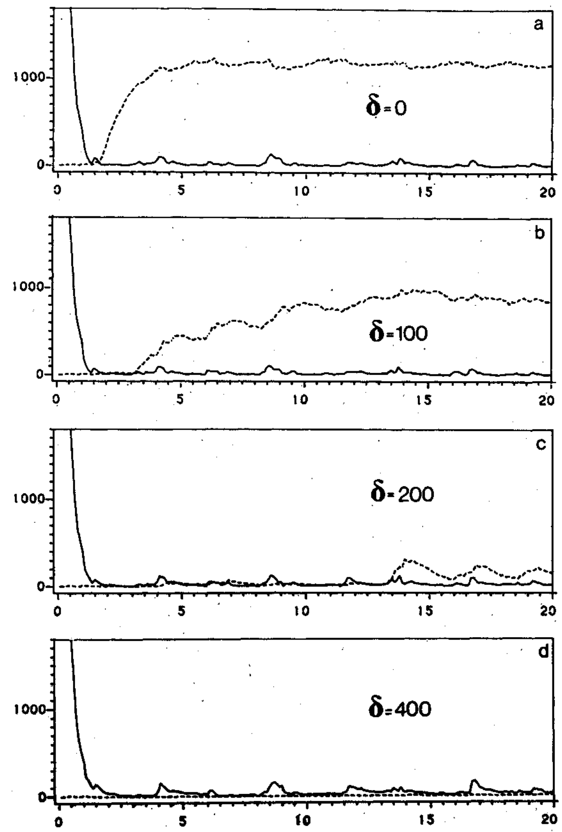

Fig. 11 Energy in controlled and residual modes for various values of δ (noise excitation, $\xi = 0.005$).

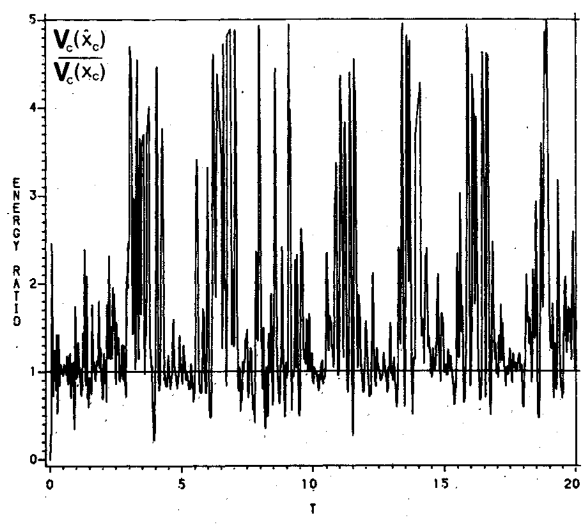

Fig. 12 Noise excitation, $\delta = 400$, $\xi = 0.005$. Time history of the ratio between the estimated and the actual energy in the controlled modes.

Noise Excitation

We consider the same example as before with the same initial conditions, but we superimpose on the control a modulated broadband noise as shown in Fig. 10. (This noise has been designed to excite the controlled as well as the residual modes.) As before, the structural damping of the residual mode is assumed to be 0.005. Figure 11 shows the energy in the controlled modes $V(x_c)$ and in the residual mode $V(x_r)$ for various values of δ. As in the previous example, with the original control ($\delta = 0$), the system goes into a limit cycle in the residual mode. As δ increases, the amplitude of the limit cycle decreases; the system is stable for $\delta = 400$ (Fig. 11d). A close examination of the full line curves in Figs. 11a-11d reveals a slight loss of performance with increasing δ. Figure 12 shows the ratio $V(\hat{x}_c)/V(x_c)$ for $\delta = 400$. The noisy shape of this curve suggests that an improved estimate of $V(x_c)$ is both feasible and advisable.

Conclusions

The application of nonlinear saturation state feedback to the control of vibration has been discussed. It has been demonstrated that, when it is based on the reconstructed state, it may lead to a strong spillover instability in the residual modes, which have small stability margins (provided by structural damping).

The following procedure to eliminate the instability has been presented. When the controlled states enter a neighborhood of the equilibrium [defined by a positive definite distance function $V(x_c)$ being smaller than a constant δ], the magnitude of the control is reduced by the ratio $V(x_c)/\delta$. The improvement is achieved without affecting the overall performance of the control, provided the estimated state $\hat{x}_c$ is close to the actual state x_c. As the discrepancy between $\hat{x}_c$ and x_c increases (which happens when the residual modes are excited), the value of δ necessary to achieve stability also increases, reducing the performance of the control system.

So far, only local stability results have been established. Further investigations will be devoted to defining the stability domain, analyzing the influence of the choice of the positive definite function $V(x_c)$, and devising an improved estimate for it.

The alleviation procedure can be used with any type of state feedback (e.g., linear).[16] It has also been shown to improve the robustness vis a vis modeling errors.[17]

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References

[1]Balas, M.J., "Trends in Large Space Structure Control Theory: Fondest Hopes, Wildest Dreams," *IEEE Transactions on Automatic Control,* AC-27, No. 3, June 1982, pp. 522-535.

[2]Balas, M.J., "Active Control of Flexible Systems," *Journal Optim. Theory and Appli.,* Vol. 25, No. 3, July 1978, pp. 415-436.

[3]Balas, M.J., "Feedback Control of Flexible Systems," *IEEE Transaction on Automatic Control,* AC-23, No. 4, Aug. 1978, pp. 673-679.

[4]Ashley, H. and Edberg, D.L., "On the Virtues and Prospects of Passive Damping in Large Shape Structures," Vibration Damping Workshop II, Las Vegas, NV, March 1986.

[5]Canavin, J., "Control Technology for Large Space Structures," AIAA Paper 78-1691, 1978.

[6]Sesak, J.R. and Coradetti, T., "Decentralized Control of Large Space Stuctures via Forced Singular Perturbation," AIAA Paper 79-0195, Jan. 1979.

[7]Sesak, J.R., Likins, R., and Coradelti, T., "Flexible Spacecraft Control by Modal Error Sensitivity Suppression," *Proceedings of VPI&SU/AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft,* Blacksburg, VA, 1979.

[8]Gupta, N.K., "Frequency-Shaped Cost Functionals: Extension of Linear-Quadratic-Gaussian Design Methods," *Journal of Guidance and Control,* Vol. 3, No. 6, Dec. 1980, pp. 529-535.

[9]Kalman, R.E. and Bertram, J.E., "Control System Analysis and Design Via the 'Second Method' of Lyapunov (1. Continuous-Time Systems)," *ASME Journal of Basic Engineering,* June 1960, pp. 371-393.

[10]Grayson, L.P., "The Status of Synthesis Using Lyapunov's Method," *Automatica,* Vol. 3, 1965, pp. 91-121.

[11]Leitmann, G., "Stabilization of Dynamical Systems under Bounded Input Disturbance and Parameter Uncertainty," *Proceedings of the 2nd Kingston Conference on Differerential Games and Control Theory,* 1976, pp. 47-64.

[12]Leitmann, G., "Guaranteed Asymptotic Stability for Some Linear Systems with Bounded Uncertainties," *ASME J. Dyn. Syst. Meas. Contr.,* Vol. 101, Sept. 1979, pp. 212-216.

[13]Leitmann, G., "Deterministic Control of Uncertain Systems," *Acta Astronautica,* Vol. 7, 1980, pp. 1457-1461.

[14]Leitmann, G., "On the Efficacy of Nonlinear Control in Uncertain Liner Systems, *ASME J. Dyn. Syst. Meas. Control,* Vol. 103, June 1981, pp. 95-102.

[15]Vidyasagar, M., *Nonlinear System Analysis,* Prentice-Hall, Englewood Cliffs, NJ, 1978.

[16]Preumont, A., "Spillover Alleviation in Active Control of Flexible Structures," *Aerospace and Ocean Engineering,* Virginia Polytechnic Institute and State Univ., Blacksburg, VA, Rept. 154, Jan. 1986.

[17]Preumont, A., "Robustness Enhancement in Active Control of Flexible Structures," Virginia Polytechnic Institute of State Univ., Blacksburg, VA, Aerospace and Ocean Engineering Rept. 155, Jan. 1986.

[18]Bingulac, S. P., West, P. J., and Perkins, W. R., "Recent Advances in the L-A-S Software used in Cad of Control Systems," *IFAC Symposium on CAD,* Copenhagen, July 1985.